THE LIFE OF A. K. ERLANG

By E. BROCKMEYER and H. L. HALSTRØM[1]).

"Bene qui latuit, bene vixit".
Ovid: Tristia, 3, 4, 25.

Agner Krarup Erlang was born on Tuesday, the 1st January, 1878, at Lønborg, a neighbouring village to the small town of Tarm situated south of the Skern rivulet in Jutland.

His father, *Hans Nielsen Erlang*, was the worthy parish clerk of the village, his official title being that of "schoolmaster and precentor". H. N. Erlang was born in South Jutland; nothing much is known about his family, but the name of Erlang is believed to be a corruption — for which German clergymen probably are responsible — of the name of Erlandsen. He had received a good education at a teachers' training college at Jelling, under the influence of *H. J. M. Svendsen*, a prominent member of the Danish folk high school, who was head of the college at the time. This college was preferred by a great number of Danish-minded students from Slesvig who did not want to receive their training at the strongly German-influenced college at Tønder.

A. K. Erlang's mother's family, on the other hand, is easier to trace. His mother was *Magdalene Krarup*, of the well-known ecclesiastical family. One of his maternal ancestors was the prominent mathematician *Thomas Fincke*, a contemporary of the great astronomer, *Tycho Brahe*; Fincke's descendants through several hundred years were holders of chairs in the University of Copenhagen. The Krarup family was, furthermore, related to the famous poet and religious leader, *N. F. S. Grundtvig*, the latter being a first cousin to Magdalene Krarup's grandmother. It was a tradition in the Krarup family that all sons should become clergymen and all daughters, clergymen's wives; and so it was a little unusual when young Magdalene chose to marry a plain village schoolmaster. But as the family got to know H. N. Erlang, they soon learnt to like and respect him very much.

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[1]) The authors are greatly indebted to the late Mr. *F. K. Erlang* and to Miss *I. Erlang* for their kindness in supplying valuable biographical information.

Agner grew up at home together with his brother, *Frederik*, by two years his senior, and his two younger sisters, *Marie* and *Ingeborg*, in not too easy circumstances. The schoolmaster's salary was very small, but he was a clever and intelligent man with an unusual talent for economizing, and his wife did not mind working hard. Therefore the family lived happily together in spite of their small income, and the children had a happy childhood and a proper upbringing. One of the father's principles was that the children should have all the food they needed, but the food must be plain, and preferably milk food; he had also decided that all milk should be carefully and thoroughly boiled so as to kill any disease-carrying germs. Although inconsistent with modern methods of preserving vitamins, this procedure was unquestionably a very sensible precaution at the time, considering the then wide-spread cattle tuberculosis.

Agner soon proved to be quick of apprehension, and he had a good memory, too; he was a quiet and peaceable boy who preferred reading to playing with the other boys. In the evenings, he and his elder brother would often share the reading of a book between them, the usual procedure being that brother Frederik would read it in the approved manner, while Agner, sitting opposite to him at the table, would read the book upside down. They got along well, as he soon learnt to read just as well in this new manner as Frederik did in his. Agner's favourite subject at that particular time was astronomy, and it was a great help that grandfather Krarup, the old vicar, had had the same hobby and had left several volumes on astronomy when he died. But Agner was not only interested in scientifical studies; the art of verse-making had also aroused his interest, and he wrote a good many poems on astronomical subjects. His poems, however, did not distinguish themselves so much by poetic flight as by strict logic. He wrote, for instance, a poem about the discovery of the planet Neptune, the first few lines of which, reproduced in English, read as follows:

"Leverrier wrote to Galle a letter,
Saying, 'Gaze at the sky you had better,
For soon up there, may be,
A planet you will see!'"

Agner spent his early school-days at his father's schoolhouse together with his brother and sisters, and when he had finished his elementary education at the village school, he stayed at home to be coached for the "Præliminæreksamen" (a sort of lower school certificate examination), partly by his father and partly, or rather chiefly, by *P. J. Pedersen*, an assistant teacher who had just been appointed to help the schoolmaster. P. J. Pedersen had

Agner Erlang at the age of 8 (right) and his brother Frederik.

graduated from a teachers' training college and was an excellent teacher; his teaching came to be of consequence to Agner Erlang's development and career. Several years later he became Mayor of Copenhagen. — In the summer of 1892, when it was fairly safe to presume that a sufficient amount of knowledge had been imparted to them, Agner and his brother

A. K. Erlang at the age of 15.

Frederik were sent to Copenhagen to enter as privately coached examinees for the "Præliminæreksamen" which was held at the University. Agner was then only 14 years old, which was below the prescribed age limit; it was therefore necessary to apply for a special entrance permission. The permission was granted, however, and Agner passed his examination with distinction.

After that, Agner returned to Lønborg and became assistant teacher at his father's school for a period of two years; at the same time he was taught

French by a teacher at the "Tarm Realskole" and Latin by the vicar of a neighbouring parish. When Agner reached the age of 16 his father thought that he ought to continue his studies and, if possible, pass the "Studentereksamen" (the University entrance examination). But money was scarce, and so Agner's father had to find out how much this might be expected to cost him. He wrote, therefore, to *M. Funch*, the county clerk of Hillerød[1]), whose wife was born Krarup, for information and advice, as their son had recently passed the "Studentereksamen". The county clerk wrote back that he had always felt indebted to the Krarup family, and would Erlang please permit him to take young Agner into his home — by way of repaying a small part of this debt — while he prepared for his examination at the Frederiksborg Grammar-School? This generous offer helped to overcome the financial difficulties in connexion with Agner Erlang's education, and he spent a happy time with the county clerk at Hillerød until he, in 1896, passed his "Studentereksamen" with distinction.

The Frederiksborg Grammar-School had the privilege of two scholarships tenable at "Regensen"[2]), and Erlang attained one of these. He now lived at "Regensen" while he studied mathematics and exact natural sciences at the University of Copenhagen. His mathematical education was greatly influenced by Prof. *H. G. Zeuthen*'s and Prof. *C. Juel*'s lectures, and all his life he maintained and cultivated his interest in geometrical problems. He finished his university studies in January, 1901, by taking the degree of candidatus magisterii (M.A.) with mathematics as principal subject and astronomy, physics, and chemistry as secondary subjects.

For some years Erlang worked as a teacher at various schools, such as: "Gammelholms Latin- & Realskole", "Femmers Kvindeseminarium" and "Lang & Hjorts Kursus" in Copenhagen, and "Vamdrup Realskole" in South Jutland. He proved to be in possession of excellent pedagogical qualities, even though his natural predilection was for scientific research rather than for teaching.

During this period, Erlang used to spend his leisure hours with a circle of young university people at "Studenterhjemmet", a Christian students' association in Copenhagen. Sympathizing with the Christian students' movement, he was for a time a member of the management of "Studenterhjemmet". He was a peaceable, not particularly sociable man who preferred to walk about as an interested spectator; this had inspired his friends to nickname him "The Private Person". Among the acquaintances he made at "Studenterhjemmet" was *H. C. Nybølle* who later was appointed professor of statistics in the University of Copenhagen. Erlang and Ny-

[1]) Market town, North Seeland, with ancient royal castle of Frederiksborg.

[2]) "Regensen" is an ancient college in Copenhagen providing free lodgings for deserving university students; it was founded in 1623 by King Christian IV.

bølle became friends for life, and they helped each other with their scientific work. Later, Nybølle's sister married A. K. Erlang's brother, *F. K. Erlang*.

In conjunction with his work as a teacher, Erlang diligently continued his studies of mathematics and natural sciences; thus, he managed to find time for attempting the mathematical prize essay of the University for 1902-1903, on Huygens' solutions of infinitesimal problems, for which attempt he was rewarded with an "accessit" in 1904. He had then already taken up the study of the theory of probabilities which later came to be the subject of his principal works.

In those years — and later — Erlang spent several of his summer vacations abroad, frequently accompanied by his brother, and a few times accompanied by *L. Christensen*, his fellow collegian who had become headmaster of the Cathedral School of Aarhus. Thus, he went to Sweden, England, Germany, and France several times. On these journeys he cultivated his many-sided interests, visiting art galleries and libraries, and he is said to have been an uncommonly interesting and pleasant travelling companion.

Erlang was a member of "Matematisk Forening" (Mathematicians' Association) and assisted regularly at its meetings. There, he became acquainted with the notable mathematician *J. L. W. V. Jensen*, Ph. D., then chief engineer and head of the technical department of the Copenhagen Telephone Company; as a result of this acquaintanceship, Erlang was introduced to the then managing director of the Company, *Fr. Johannsen*, D. Sc.

The merit of having introduced the theory of probabilities into telephony is due to Dr. Johannsen as he had published, in 1907 and 1908, two essays under the respective titles of "Waiting Times and Number of Calls"[1]) and "Busy"[2]), the former dealing with the delay problem in connexion with incoming calls in manual telephone exchange switchboards, and the latter being an investigation as to how often subscribers with one or more lines each are reported "busy". In both papers, the problems were coped with by means of the theory of probabilities; true, the method was not mathematically exact, but the results obtained were sufficiently correct to be serviceable for practical purposes.

Dr. Johannsen, who had a remarkable talent for selecting assistants of the right sort, suggested to Erlang that he should take these problems up for mathematical treatment. For some time Dr. Johannsen had been

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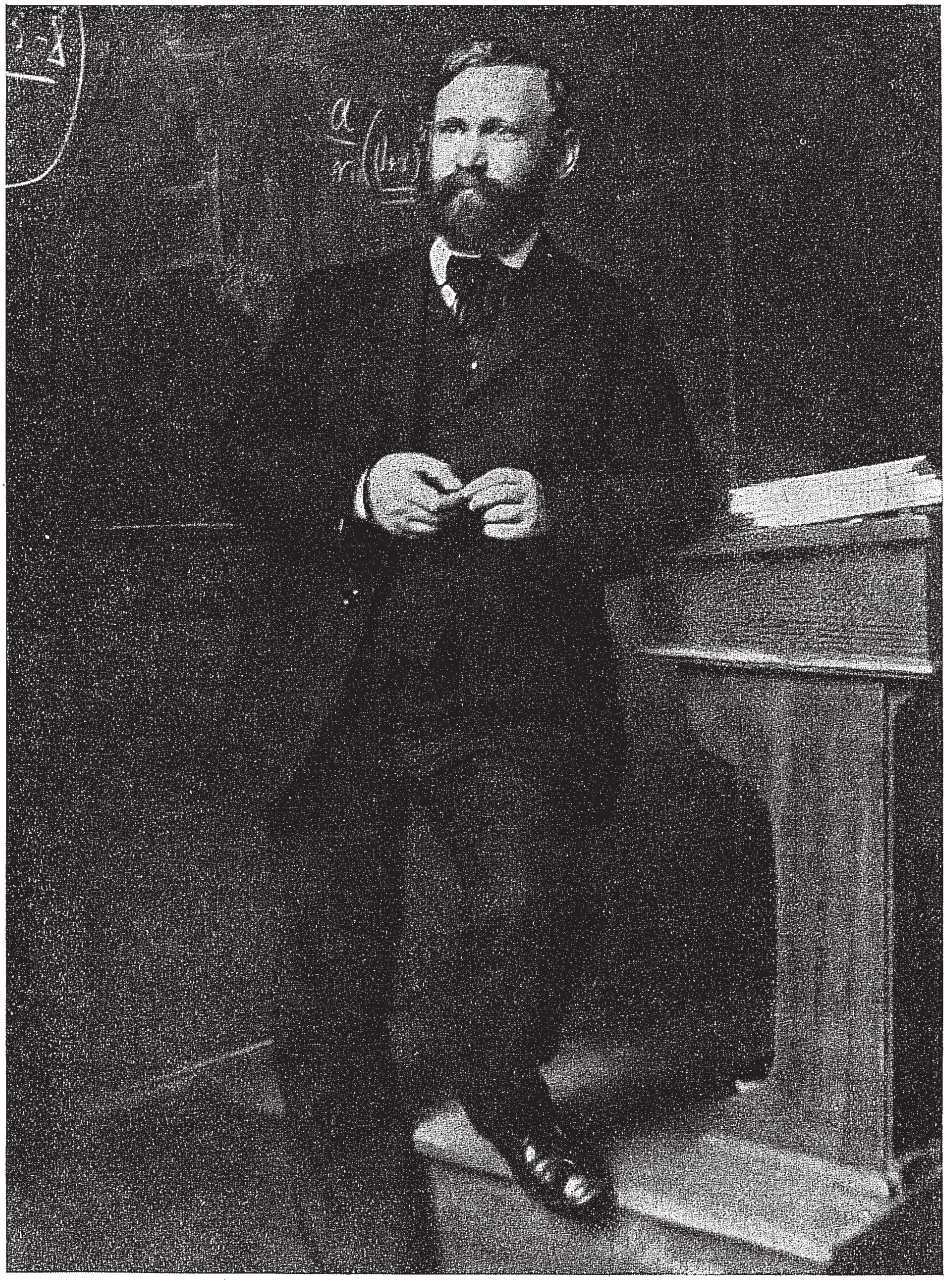
[1]) Reprinted in "Telephone Management in Large Cities", Post Office Electrical Engineers Journal, London, Oct. 1910 and Jan. 1911.

[2]) Reprinted in "The Development of Telephonic Communication in Copenhagen 1881-1931", Ingeniørvidenskabelige Skrifter A, No. 32, Copenhagen, 1932, p. 150.



A. K. Erlang, 32 years old.

nursing the idea of establishing a physico-technical laboratory for scientific research work, as the development and importance of the Copenhagen Telephone Company seemed to justify such a step, and so, in 1908, the Company engaged Erlang as scientific collaborator and head of its laboratory. In this position, which gave him an opportunity to develop and utilize his great gifts and considerable knowledge, he worked for the rest of his life.

With enthusiasm and diligence, Erlang immediately set to work at applying the theory of probabilities to problems of telephone traffic, the domain that was to make his name widely known. As early as in 1909 he published his first work on this subject, "The Theory of Probabilities and Telephone Conversations", in which he proved that telephone calls distributed at random follow the Poisson law of distribution, and gave the exact solution of the delay problem stated in Dr. Johannsen's essay of 1907, in the special case of only one operator being available to handle the calls.

Erlang also cooperated with Dr. J. L. W. V. Jensen's successor-to-be as chief engineer to the Company, *P. V. Christensen*, who in 1913 published his paper on "The Number of Selectors in Automatic Telephone Exchanges"[1]) in which he, as the first, treated these problems by means of the theory of probabilities. Erlang contributed to this paper by preparing the tables stating the probability of loss.

In 1917 Erlang published his most important work, "Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges", containing his formulae for loss and waiting time which he had developed on the basis of the principle of statistical equilibrium; these now well-known formulae are of fundamental importance to the theory of telephone traffic.

In the course of the years, Erlang published several other valuable works on the theory of telephone traffic and some smaller works with reference to other mathematical domains, especially the calculation of tables of logarithms and other numerical tables. All these shall not be enumerated here, however, as a survey of Erlang's works is given on pp. 101-130.

Nearly all his works have first been published in Danish in the form of articles in various journals, but the most important of them have later been translated into one or more foreign languages such as English, French, and German, and printed in foreign journals. Erlang had a decided propensity to concise speech, and he wrote his essays in a very brief style, too; in fact, his conciseness was so ingrained that he even published many

[1]) Published, in "Elektroteknikeren", 1913, p. 207, also in Elektrotechnische Zeitschrift, 1913, p. 1314 and in Post Office Electrical Engineers Journal, 1914, p. 271.

of his results without giving the proofs. The concise style and the omission of the proofs serve, to some extent, to complicate the study of his original works for readers who are not specialists in the relevant domains.

However, Erlang's works on the theory of telephone traffic soon won recognition and understanding not only in the Scandinavian countries, but also in other countries. Thus, a few years after its appearance his formula for the probability of loss was accepted by the British Post Office as basis for calculations respecting circuit facilities. It may be mentioned as an example of the interest taken in his works that two researchers of the subject taught themselves Danish in order to be able to read Erlang's papers in the original language, namely Dr. *A. E. Vaulot*, a Frenchman who has translated some of Erlang's works into French, and Dr. *Thornton C. Fry* of the Bell Telephone Laboratories, U. S. A.

As the leader of the laboratory of the Telephone Company, Erlang had opportunity to grapple with numerous and varied physico-technical problems, one of his first being the measuring of stray currents. Prof. *Absalon Larsen* and *S. A. Faber* had previously laid the foundation by investigating the distribution of stray currents and their damaging influence upon the lead sheaths of telephone cables, but it fell to Erlang's lot to systematize the practical procedure. At first Erlang had no laboratory staff to assist him; he had to carry out all measurements of stray currents in person, and so he could be seen frequently in the streets of Copenhagen followed by a workman carrying a ladder, which was used for the purpose of climbing down into the manholes.

Erlang published some works dealing with various problems pertaining to the physico-technical side of telephony that were of considerable importance at the time of their appearance. For the purpose of measuring alternating currents, for instance, he constructed a measuring apparatus — the so-called "Erlang's Complex Compensator" — which represented a considerable improvement, compared with similar types of measuring instruments of earlier date. He did not publish many works of this kind, however, as most of his laboratory tasks consisted in the solution of concrete problems in connexion with loaded cables, transmission schemes, etcetera. A great part of these practical results of his would undoubtedly have been of interest to many, but Erlang used to think that he could not afford the time it would be necessary to spend on preparing such works for publication.

The engineers of the Telephone Company could always rely on Erlang when they needed his help in questions of physical or mathematical nature; a frequent visitor to the public libraries, he had an amazing knack of procuring literature to cover any topical matter. Erlang had his own, almost Socratic, manner of answering questions put to him: as a rule,

he would hesitate to give a solution of the question asked, directly, preferring instead to enter into a sometimes lengthy discussion that would elucidate the subject from any conceivable point of view; in this manner he forced the inquirer into thinking the matter over on his own, thereby perhaps finding his way to solve the problem independently. After such a discussion with Erlang one always felt enriched far beyond the scope of its original subject.

Erlang remained single all his life. There was a time, however, when he was very much interested in a pretty young girl; but she married one of Erlang's fellow collegians, and it took him a long time to live down his disappointment. He was fond of children, and they liked him; he enjoyed to chat with children, and sometimes even taught them to play chess.

Erlang devoted all his energy to his scientific studies and his work at the laboratory of the Telephone Company. He would often work far into the night in his study at home, and when he eventually retired to bed, he armed himself with four or five books to choose between; then, he would open one of the books and begin to read, usually falling asleep and forgetting about his book and the electric light a few minutes later. When his sister discovered that the light was burning to no purpose at all, she sometimes tip-toed into his room and turned it off; it never failed, on these occasions, that he immediately woke up and exclaimed in a voice which he tried hard to make severe and offended, "I am working!" — whereupon his sister hurriedly withdrew, of course.

Erlang collected a library — a remarkably large one, at that, for a private person — chiefly comprising works about mathematics, physics, and astronomy. His knowledge of these subjects was great and comprehensive, and yet he was interested in a good many other subjects as well, such as philosophy, history, and poetry; he had a special liking for *Pascal* and his production.

Besides being a member of "Matematisk Forening", the meetings of which he attended regularly, Erlang was an associate of the British Institution of Electrical Engineers.

Erlang had a noteworthy and original personality. He was a sincere Christian in a sympathetic way, at the same time being full of humour and satirical wit; outwardly, his heavy red full beard and his manner of dressing lent a certain artistic touch to his characteristic appearance. Extremely modest and unobtrusive of demeanour, he preferred the peaceful atmosphere of his study to social gatherings and festivities; he never touched alcoholic liquors nor smoked tobacco. His mode of life has been expressed very appropriately in the words which Prof. Nybølle put as the motto of his obituary[1]) of Erlang, his intimate friend: the well-known

[1]) Published in "Matematisk Tidsskrift" B, 1929, p. 32.

quotation which we have prefixed to this biography, too. The recognition and esteem that fell to Erlang's lot, especially towards the end of his life, quite naturally made him happy, though.

Erlang was a beneficent man; living frugally, he could afford to help others, which he did to an even very great extent. His youngest sister, Miss *Ingeborg Erlang*, with whom he shared apartments for many years right up to his death, had founded a home for feeble-minded women with her own money, and in the course of the years Erlang donated a considerable part of his income to this home. Needy people often turned up at the laboratory to apply to Erlang for help, and he would invariably help them with ready money in as inconspicuous a manner as possible.

For a period of nearly 20 years Erlang had served the Copenhagen Telephone Company without a single day's absence on account of illness. In January, 1929, he felt ill, however, and it turned out that he was suffering from an abdominal disease that would necessitate an operation. Before he was put in hospital, he took leave of all his colleagues at the Company, telling them that he was going to stay in hospital for a short time only; but when the tidings of his death a few days later reached the Telephone House, his friends realized that he must have known his time was drawing near. His religious conviction and his philosophical mode of thought had bestowed upon him the serenity with which he went to meet his death.

Agner Krarup Erlang died on Sunday, the 3rd February, 1929, only 51 years old.

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In the autumn of 1943, the editors of the Swedish journal "Tekniska Meddelanden från Kungl. Telegrafstyrelsen" invited — in connexion with an essay[1]) by *Conny Palm*, Sc. D. — interested parties to enter for a prize competition to be held for the purpose of finding a suitable name for the natural unit of telephone traffic. This unit had not hitherto had any particular name, for which reason such a vague denomination as "Traffic Unit" (and in other languages, correspondingly, "Trafikenhed", "Verkehrseinheit", etcetera) had been used, which might easily give occasion to confusion with other units of telephone traffic being employed in practice, for instance: *Sm*, "Speech minutes"; *E. B. H. C.* "Equated Busy Hour Calls".

In Danish quarters this gave rise to the idea of identifying A. K. Erlang's name with the natural unit of traffic. Accordingly, Mr. *P. V. Christensen*, Chief Engineer to the Copenhagen Telephone Company; Mr. *N. E. Holmblad*, Chief Engineer to the Danish Post & Telegraph Office; Prof.

[1]) *Conny Palm:* Samtalsminut eller Trafikenhet, T. M. f. K. T., 1943, p. 133.

J. Oskar Nielsen, and Prof. *J. Rybner*, both of the Royal Danish College of Engineering, sent the following note to the editors of "Tekn. Medd. fr. Kungl. Telegrafstyrelsen":

"In connexion with Dr. Conny Palm's essay: "Speech Minute or Traffic Unit" in "Tekniska Meddelanden", 1943, nos. 7-9, and the editors' invitation to enter for a prize competition with the object of finding a new name for the natural unit of telephone traffic, we wish to express our sympathy with the idea of introducing a suitable name that is well adapted to be generally approved for international application.

"We would like to connect the said unit of traffic with the name of the prominent researcher of the theory of telephone traffic, Magister *A. K. Erlang* (1878-1929), whose pioneer works in this field are universally known and appreciated. It is sufficient to mention the formula, published by Erlang in 1917, for the probability of loss in the case of a simple group of circuits, the so-called "B-formula", which must be regarded as the first mathematically exact solution of a problem of barred access; in this formula enters, besides the number of circuits (x), the intensity of traffic (y) as expressed in terms of the said natural unit of traffic.

"We beg to suggest, therefore, that the name of "*Erlang*" be introduced as the denomination of the natural unit of traffic.

"The suggested denomination is in analogy with well-known unit denominations such as Ohm, Ampère, Gauss, Maxwell, Örsted, etcetera.

"It would give us great pleasure if the adjudication committee would support our suggestion by accepting the name of "Erlang" as the denomination of the unit of traffic, thereby contributing to the introduction of this name into telephony."

The result of the competition was that the committee accepted the name of "Erlang" which, by the way, had also been suggested by 8 out of the 26 Swedish competitors. The adjudication committee gave the following three reasons for its decision:

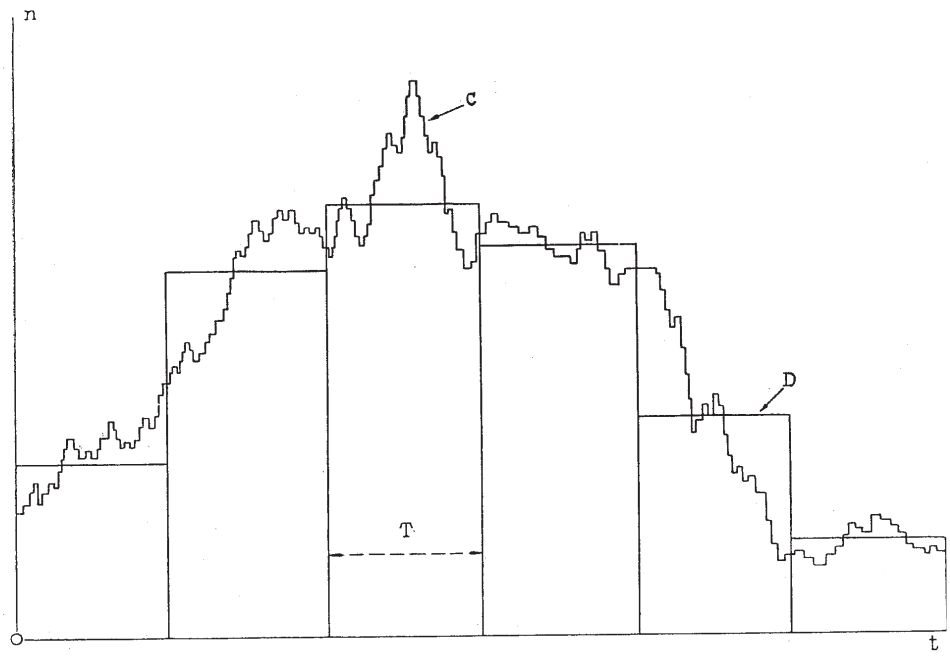
1) "Erlang" brings the name of the founder of modern telephone traffic research to memory.

2) "Erlang" seems phonetically attractive, which is of consequence to a possible international acceptance of the name.

3) "Erlang" is not formed by abbreviation of any term in any specific language, which means the absence of any risk of metamorphosis by translation into any other language.

In consequence of this decision the name of "Erlang" has been used in the Scandinavian countries to denominate the unit of traffic since the

traffic intensity is usually involved in calculations of the number of connecting devices, so that the step-like diagram D (shown in the figure) is used instead of the curve C. Each of the rectangles in this diagram has the same area as that bounded by the corresponding segment of the



curve. The height of each rectangle is equal to the quantity, expressed in "erlang", that characterizes the average intensity of traffic during the period represented by the base of the rectangle; if total occupancy of all connecting devices never occurs during the period T, the said quantity in terms of "erlang" also expresses the amount of traffic to be handled.

"The name of "erlang" is accepted in recognition of the important investigations of the applicability of the theory of probabilities to telephonic problems that were carried out by the Danish scientist *Erlang*."